Mechanical systems with external forces. Symmetries, reduction and Hamilton-Jacobi theory

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Abstract

Many physical systems require an external force, besides the Hamiltonian or Lagrangian function, to describe their dynamics. Moreover, external forces arise after performing a process of reduction in a nonholonomic system with symmetries. Geometrically, external forces can be characterized as semibasic 1-forms on a symplectic manifold (usually, the phase space T^*Q or the space of velocities TQ). In our work, we extend significant results from geometric mechanics, such as Noether-like theorems, symplectic reduction or Hamilton-Jacobi theory, to systems with external forces.

In a first paper [1], we have obtained a Noether's theorem for Lagrangian systems with external forces. We have also extended other results regarding symmetries and constants of the motion for systems with external forces. Our results are particularized for the so-called Rayleigh dissipation (i.e., an external force that can be written as the derivative of a "potential" with respect to the velocities). Furthermore, we present a theory for the reduction of Lagrangian systems with external forces, in which both the Lagrangian function and the external force are invariant under a Lie (sub)group of symmetries.

In a second paper [2], currently under preparation, we generalize the results from Hamilton-Jacobi theory previously obtained for conservative systems. Geometrically, the solutions of the Hamilton-Jacobi problem are described as sections of TQ or T^*Q, which allows to extend it for systems with external forces in a natural manner. We study the complete solutions for the Hamilton-Jacobi problem and present some examples. Moreover, we present a discrete Hamilton-Jacobi theory for systems with external forces.

References

[1] M. de León, M. Lainz, and A. López-Gordón, *"Symmetries, constants of the motion, and reduction of mechanical systems with external forces"*, J. Math. Phys. **62**, 042901 (2021)

[2] M. de León, M. Lainz, and A. López-Gordón, *"Geometric Hamilton-Jacobi theory for systems with external forces and discretization"*, under preparation