Curvature-adapted submanifolds of bi-invariant Lie groups

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Abstract

Given a hypersurface M of a Riemannian manifold Q, one says that M is *curvature-adapted* (*to* Q) if, for each $p \in M$, the normal Jacobi operator and the shape operator of M commute. The first operator measures the curvature of the ambient manifold along the normal vector of M, whereas the second describes the curvature of M as a submanifold of Q. This condition can be generalized to submanifolds of arbitrary codimension.

In this talk, we will study curvature-adapted submanifolds in a Lie group G equipped with a bi-invariant Riemannian metric. In particular, we shall see that, if the normal bundle of $M \subset G$ is *abelian* [3] (for every $p \in M$, $\exp(N_pM)$ is contained in some totally geodesic, flat submanifold of G), then any normal Jacobi operator of M equals the square of the corresponding *invariant shape operator* [2]. This permits to understand curvature-adaptedness to G in terms of left translations [1, Theorem 1].

For example, it turns out that, in the case where M is a hypersurface, the normal Jacobi operator commutes with the ordinary shape operator precisely when the left-invariant extension of each of its eigenspaces remains tangent to M along all the others. As a further consequence of the same result, one observes that any surface in a three-dimensional bi-invariant Lie group is curvature-adapted.

References

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