Lagrangian reduction by fibered actions in Field Theory

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Abstract

In the realm of geometric mechanics and field theory, reduction consists of taking advantage of the symmetries of a system to simplify both the geometry of the problem and the dynamical equations. More specifically, given a Lagrangian theory, we can perform the quotient of the configuration space (or configuration bundle, in the case of field theories) by the group of symmetries. Then the principle of stationary action is transferred to this *reduced* space, thus yielding the reduced equations. This ideas have been largely developed in mechanics, where the so-called Euler-Poincaré and Lagrange-Poincaré equations are obtained [2]. Likewise, these equations have their counterpart in field theory [1, 3].

For field theories the case of global symmetries is the only one that has been treated so far, that is, when the group of symmetries acting on the configuration bundle is the same for every point of the base space. Nevertheless, there are many physical systems that have local symmetries, i.e. the group of symmetries depends of the point of the base space. Mathematically, a local symmetry is given by a fibered action of a Lie group bundle on the corresponding configuration bundle of the theory.

In this talk we explore the reduction procedure for a Lagrangian field theory with local symmetries and its applications to gauge theories. In this case, the reduction can be only performed partially, meaning that some of the unreduced variables still appear in the reduced equations. This agrees with the well known Utiyama's theorem, where the unreduced connection (physically, the potential) 'survives' in the Yang-Mills equations.

References

[1] M. Castrillón, P. García, T. Ratiu: *Euler-Poincaré reduction on principal bundles.* Lett. Math. Phys.: **58**, 167–180 (2001).

[2] H. Cendra, J. Marsden, T. Ratiu: *Lagrangian reduction by stages.* Mem. Amer. Soc. V: **152**, (2001).

[3] D. Ellis, F. Gay-Balmaz, D. Holm, T. Ratiu: *Lagrange-Poincaré field equations.* J. Geom. Phys. **61**, 2120–2146 (2011).