Mechanical nonholonomic trajectories are Riemannian geodesics!

Alexandre Anahory Simões[a], Juan Carlos Marrero and David Martín de Diego

[a] ICMAT (CSIC-UAM), Calle Nicolas Cabrera, 13-15, 28049, Madrid (Spain)
E-mail: alexandre.anahory@icmat.es
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Abstract

We will present a new and surprising result in nonholonomic mechanics. In a nutshell, nonholonomic mechanics governs the motion of systems with constraints on velocity. One of its most remarkable properties is that the derivation of the nonholonomic equations is not variational, tearing apart unconstrained and nonholonomic dynamics. However, in this talk, we will show that mechanical nonholonomic systems may be seen as variational if we choose an appropriate Riemannian structure. In fact, we will show slightly more: we show that trajectories are geodesics relative to this structure so that, in particular, they are length minimizing!

To achieve this goal, we will give a short introduction to nonholonomic mechanics: constraints on velocities, equations of motion obtained from a Lagrangian function and examples of real life nonholonomic systems. Next, we will consider kinetic nonholonomic systems, which are derived from a distribution $\mathcal{D}$ and a kinetic Lagrangian of the form $L_g(v) = \frac{1}{2}g(v, v)$. We will present the construction of the nonholonomic exponential map at some fixed point q which identifies the vector space at q in the distribution with a submanifold of the ambient manifold containing q. This map is the key to understand our main theorem (cf. [1]). We will define a Riemannian structure on this submanifold. Moreover, we show that the solutions starting at q evolve on this submanifold and are true geodesics relative to the new Riemannian structure. Then we present a nonholonomic version of the famous Maupertuis principle (see [2]) which states that trajectories of mechanical nonholonomic systems with fixed energy e are reparametrizations of trajectories of the kinetic nonholonomic system associated to the Jacobi metric.

Finally, we will discuss some nice juicy future applications of these results both on theoretical as well as on applied problems. This is joint work with David Martin de Diego and Juan Carlos Marrero.

References

[1] Anahory Simões, A., Marrero, J. C., Martín de Diego, D.: *Radial kinetic nonholonomic trajectories are Riemannian geodesics!*. arXiv:2010.12444, 2020.

[2] Anahory Simões, A., Marrero, J. C., Martín de Diego, D.: *Contact bundle formulation of nonholonomic Maupertuis-Jacobi principle and a length minimizing property of nonholonomic dynamics*. arXiv:2104.13178v1, 2021.